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Weak forms of continuity in *I*-double gradation fuzzy topological spaces

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Abstract

In this paper, we introduce and characterize double fuzzy weakly preopen and double fuzzy weakly preclosed functions between *I*-double gradation fuzzy topological spaces and also study these functions in relation to some other types of already known functions.

Introduction

In the history of science, new theories have always been necessary in order for existing scientific theories to progress and this will continue to be true in the future. Two examples of essentially different mathematical theories that deal with the concept of uncertainty are probability theory and the theory of fuzzy sets. Whereas probability theory has a history of around 360 years, the theory of fuzzy sets is little more than 50 years old. Since the 1960s fuzzy methods have entered the scientific and technological world, good theoretical progress (e.g., fuzzy logic, fuzzy probability theory, fuzzy topology, fuzzy algebra) has been made, and there have been technical advances in various areas (e.g., fuzzy control, fuzzy expert systems, fuzzy clustering and data mining).

Chang (1968); Lowen (1976); Šostak (1985); Kubiak (1985); Samanta and Mondal () and many others contributed a lot to the field of Fuzzy Topology. In recent years Fuzzy Topology has been found to be very useful in solving many practical problems. Shihong Du et. al. (2005) are currently working to fuzzify the 9-intersection Egenhofer model Egenhofer and Franzosa (1991); Herring and Egenhofer (1991) for describing topological relations in Geographic Information Systems (GIS) query. In El-Naschie (1998, 2000), El-Naschie has shown that the notion of Fuzzy Topology is applicable to quantum particle physics and quantum gravity in connection with String Theory and e^{∞} Theory. Tang (2004) has used a slightly changed version of Chang's fuzzy topological space to model spatial objects for GIS databases and Structured Query Language (SQL) for GIS.

In this paper, we will introduce the concepts of double fuzzy weakly preopen and double weakly preclosed functions in I-double gradation fuzzy topological spaces. Their properties and the relationships between these functions and other functions introduced previously are investigated.

Preliminaries

Throughout this paper, let X be a nonempty set and I is the closed unit interval $[0,1]$. $I_{\circ} = (0,1]$ and $I_1 = [0,1)$. The family of all fuzzy subsets on X denoted by I^X. By $\underline{0}$ and $\underline{1}$, we denote the smallest and the greatest fuzzy subsets on X. For a fuzzy subset $\lambda \in I^X$, $\underline{1} - \lambda$ denotes its complement. Given a function $\tilde{f} : X \to Y$, $f(\lambda)$ and $f^{-1}(\lambda)$ define the direct image and the inverse image of f, defined by $f(\lambda)(y) = \bigvee_{f(x)=y} \lambda(x)$ and $f^{-1}(\nu)(x) = \nu(f(x))$, for each $\lambda \in I^X$, $\nu \in I^Y$, and $x \in X$, respectively. For fuzzy subsets λ and μ in X, we write $\lambda q \mu$ to mean that λ is quasi coincident (q-coincident) with μ, that is, there exists at least one point $x \in X$ such that $\lambda(x) + \mu(x) > 1$. Negation of such a statement is denoted as $\lambda \bar{q} \mu$. Notions and notations not described in this paper are standard and usual.

Definition 2.1. [(Samanta and Mondal (1997, 2002); Garcia and Rodabaugh (2005)] *An I-double gradation fuzzy topology* (τ, τ^*) *on* X *is a pair of maps* $\tau, \tau^* : I^X \to I$, *which satisfies the following properties:*

(O1) $\tau(\lambda) \leq \underline{1} - \tau^*(\lambda)$ *for each* $\lambda \in I^X$.
(O2) $\tau(\lambda_1 \wedge \lambda_2) \geq \tau(\lambda_1) \wedge \tau(\lambda_2)$ *and* $\tau^*(\lambda_1 \wedge \lambda_2) \leq \tau^*(\lambda_1) \vee \tau^*(\lambda_2)$ *for each* $\lambda_1, \lambda_2 \in I^X$.

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(O3) $\tau(\bigvee_{i\in\Gamma}\lambda_i) \geq \bigwedge_{i\in\Gamma}\tau(\lambda_i)$ *and* $\tau^*(\bigvee_{i\in\Gamma}\lambda_i) \leq \bigvee_{i\in\Gamma}\tau^*(\lambda_i)$ *for each* $\lambda_i \in I^X$, $i \in \Gamma$.

The triplet (X, τ, τ^*) is called an I-double gradation fuzzy topological spaces (I-dfts, for short). A fuzzy set λ is called an (r,s)-fuzzy open ((r,s)-fo, for short) if $\tau(\lambda) \geq r$ and $\tau^*(\lambda) \leq s$. A fuzzy set λ is called an (r,s)-fuzzy closed ((r,s)-fc, for short) set iff $\underline{1} - \lambda$ is an (r,s)-fo set. Let (X, τ_1, τ_1^*) and (Y, τ_2, τ_2^*) be two I-dfts's. A function $\tilde{f} : X \to Y$ is said to be a double fuzzy continuous iff $\tau_1(f^{-1}(\nu)) \geq \tau_2(\nu)$ and $\tau_1^*(f^{-1}(\nu)) \leq \tau_2^*(\nu)$ for each $\nu \in I^Y$.

There was a question we must ask ourselve before starting to present our results, which was: *Is it useful to introduce new concepts to I-double gradation fuzzy topological spaces*?

We could know that double (initially, intuitionistic) fuzzy sets (and hence double fuzzy topological spaces) deal with ambiguity in a way better than fuzzy sets. In addition to that, double fuzzy topological spaces is a generalization of some other kinds of topological spaces; we can get fuzzy topological spaces in Chang's sense $(X, \mathcal{T}_{r,s})$, where

$$\mathcal{T}_{(r,s)} = \{\lambda \in I^X | \tau(\lambda) \geq r, \quad \tau^*(\lambda) \leq s\}.$$

Also, when the conditions $\tau^*(\lambda) = 1 - \tau(\lambda)$ and $\tau(\lambda) + \tau^*(\lambda) \not\leq 1$ achieved in Definition 2.1, we get the definition of fuzzy topological spaces in Kubiak- Šostak's sense Kubiak (1985); Šostak (1985). If we use 2^X instead of I^X, the resulting topological structure will be called double gradation fuzzifying topological spaces (A new structure mentioned for the first time in Bhaumik and Abbas 2008). Besides, we can also get the general topological spaces.

Theorem 2.1. [(Çoker and Demirci 1996; Lee and Im (2001)] *Let* (X, τ, τ^*) *be an I-dfts. Then for each* $r \in I_0$, $s \in I_1$ *and* $\lambda \in I^X$, *we define an operator* $C_{\tau,\tau^*} : I^X \times I_0 \times I_1 \to I^X$ *as follows:*

$$C_{\tau,\tau^*}(\lambda, r, s) = \bigwedge\{\mu \in I^X \mid \lambda \leq \mu,\ \tau(\underline{1}-\mu) \geq r,\ \tau^*(\underline{1}-\mu) \leq s\}.$$

For λ, $\mu \in I^X$, $r_1, r_2 \in I_0$ *and* $s_1, s_2 \in I_1$, *the operator* C_{τ,τ^*} *satisfies the following statements:*

(C1) $C_{\tau,\tau^*}(\underline{0}, r, s) = \underline{0}$,
(C2) $\lambda \leq C_{\tau,\tau^*}(\lambda, r, s)$,
(C3) $C_{\tau,\tau^*}(\lambda, r, s) \vee C_{\tau,\tau^*}(\mu, r, s) = C_{\tau,\tau^*}(\lambda \vee \mu, r, s)$,
(C4) $C_{\tau,\tau^*}(\lambda, r_1, s_1) \leq C_{\tau,\tau^*}(\lambda, r_2, s_2)$ *if* $r_1 \leq r_2$ *and* $s_1 \geq s_2$,
(C5) $C_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda, r, s), r, s) = C_{\tau,\tau^*}(\lambda, r, s)$.

Theorem 2.2. [(Çoker and Demirci 1996; Lee and Im 2001)] *Let* (X, τ, τ^*) *be an I-dfts. Then for each* $r \in I_0$, $s \in I_1$ *and* $\lambda \in I^X$, *we define an operator* $I_{\tau,\tau^*} : I^X \times I_0 \times I_1 \to I^X$ *as follows:*

$$I_{\tau,\tau^*}(\lambda, r, s) = \bigvee\{\mu \in I^X \mid \mu \leq \lambda,\ \tau(\mu) \geq r,\ \tau^*(\mu) \leq s\}.$$

For $\lambda, \mu \in I^X$, $r, r_1, r_2 \in I_0$ *and* $s, s_1, s_2 \in I_1$, *the operator* I_{τ,τ^*} *satisfies the following statements:*

(I1) $I_{\tau,\tau^*}(\underline{1} - \lambda, r, s) = \underline{1} - C_{\tau,\tau^*}(\lambda, r, s)$,
(I2) $I_{\tau,\tau^*}(\underline{1}, r, s) = \underline{1}$,
(I3) $I_{\tau,\tau^*}(\lambda, r, s) \leq \lambda$,
(I4) $I_{\tau,\tau^*}(\lambda, r, s) \wedge I_{\tau,\tau^*}(\mu, r, s) = I_{\tau,\tau^*}(\lambda \wedge \mu, r, s)$,
(I5) $I_{\tau,\tau^*}(\lambda, r_1, s_1) \geq I_{\tau,\tau^*}(\lambda, r_2, s_2)$ *if* $r_1 \leq r_2$ and $s_1 \geq s_2$,
(I6) $I_{\tau,\tau^*}(I_{\tau,\tau^*}(\lambda, r, s), r, s) = I_{\tau,\tau^*}(\lambda, r, s)$,
(I7) *If* $I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda, r, s), r, s) = \lambda$, *then* $C_{\tau,\tau^*}(I_{\tau,\tau^*}(\underline{1} - \lambda, r, s), r, s) = \underline{1} - \lambda$.

Definition 2.2. *Let* (X, τ, τ^*) *be an I-dfts. For* $\lambda \in I^X$, $r \in I_0$ *and* $s \in I_1$.

(1) *λ is called (r,s)-fuzzy preopen ((r,s)-fpo, for short) if $\lambda \leq I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda, r, s), r, s)$. A fuzzy set λ is called (r,s)-fuzzy preclosed ((r,s)-fpc, for short) iff $\underline{1} - \lambda$ is (r,s)-fpo set. The (r,s)-fuzzy preinterior of λ, denoted by $PI_{\tau,\tau^*}(\lambda, r, s)$ is defined by*

$$PI_{\tau,\tau^*}(\lambda, r, s) = \bigvee\{\nu \in I^X \mid \nu \leq \lambda,\ \nu \text{ is } (r,s)-fpo\}.$$

The (r,s)-fuzzy preclosure of λ, denoted by $PC_{\tau,\tau^*}(\lambda, r, s)$ is defined by

$$PC_{\tau,\tau^*}(\lambda, r, s) = \bigwedge\{\nu \in I^X \mid \lambda \leq \nu,\ \nu \text{ is } (r,s)-fpc\}.$$

(2) *λ is called (r,s)-fuzzy regular open ((r,s)-fro, for short) if $\lambda = I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda, r, s), r, s)$. A fuzzy set λ is called (r,s)-fuzzy regular closed ((r,s)-frc, for short) iff $\underline{1} - \lambda$ is (r,s)-fro set.*
(3) *λ is called (r,s)-fuzzy α-open ((r,s)-fαo, for short) if $\lambda \leq I_{\tau,\tau^*}(C_{\tau,\tau^*}(I_{\tau,\tau^*}(\lambda, r, s), r, s), r, s)$. A fuzzy set$\lambda$ is called (r,s)-fuzzy α-closed ((r,s)-fαc, for short) iff $\underline{1} - \lambda$ is (r,s)-fαo set.*

Theorem 2.3. *Let* (X, τ, τ^*) *be an I-dfts. For* $\lambda \in I^X$, $r \in I_0$ *and* $s \in I_1$.

(1) *λ is (r,s)-fpo (resp. (r,s)-fpc) iff $\lambda = PI_{\tau,\tau^*}(\lambda, r, s)$ (resp. $\lambda = PC_{\tau,\tau^*}(\lambda, r, s)$),*
(2) $I_{\tau,\tau^*}(\lambda, r, s) \leq PI_{\tau,\tau^*}(\lambda, r, s) \leq \lambda \leq PC_{\tau,\tau^*}(\lambda, r, s) \leq C_{\tau,\tau^*}(\lambda, r, s)$,
(3) $\underline{1} - PI_{\tau,\tau^*}(\lambda, r, s) = PC_{\tau,\tau^*}(\underline{1} - \lambda, r, s)$ *and* $PI_{\tau,\tau^*}(\underline{1} - \lambda, r, s) = \underline{1} - PC_{\tau,\tau^*}(\lambda, r, s)$.

Definition 2.3. *Let* $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ *be a function from an I-dfts* (X, τ_1, τ_1^*) *into an I-dfts* (Y, τ_2, τ_2^*). *The function f is called:*

(1) *double fuzzy preclosed if* $f(\lambda)$ *is* (r,s)*-fpc set in* I^Y *for each* $\lambda \in I^X$, $r \in I_0$ *and* $s \in I_1$; $\tau_1(\underline{1}-\lambda) \geq r$, $\tau_1^*(\underline{1}-\lambda) \leq s$,
(2) *double fuzzy open if* $\tau_2(f(\lambda)) \geq \tau_1(\lambda)$ *and* $\tau_2^*(f(\lambda)) \leq \tau_1^*(\lambda)$ *for each* $\lambda \in I^X$, $r \in I_0$ *and* $s \in I_1$,
(3) *double fuzzy almost open if* $\tau_2(f(\lambda)) \geq r$ *and* $\tau_2^*(f(\lambda)) \leq s$ *for each* (r,s)*-fro set* $\lambda \in I^X$, $r \in I_0$ *and* $s \in I_1$.

Definition 2.4. *Let* $f : (X, \tau_1, \tau_1^*) \rightarrow (Y, \tau_2, \tau_2^*)$ *be a function from an I-dfts* (X, τ_1, τ_1^*) *into an I-dfts* (Y, τ_2, τ_2^*). *The function f is called:*

(1) *double fuzzy weakly open if* $f(\lambda) \leq I_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\lambda, r, s)), r, s)$ *for each* $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$; $\tau_1(\lambda) \geq r$ and $\tau_1^*(\lambda) \leq s$,
(2) *double fuzzy* α*-open if* $f(\lambda)$ *is* (r,s)*-fαo in* I^Y *for each* $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$; $\tau_1(\lambda) \geq r$ and $\tau_1^*(\lambda) \leq s$.

Definition 2.5. *Let* (X, τ, τ^*) *be an I-dfts,* $\mu \in I^X$, $x_t \in \mathbf{P}(X)$, $r \in I_0$ *and* $s \in I_1$ *where* $\mathbf{P}(X)$ *is the family of all fuzzy points in X.* μ *is called an* (r,s)*-fuzzy open Q-neighborhood of* x_t *if* $\tau(\mu) \geq r$, $\tau^*(\mu) \leq s$ *and* $x_t q \mu$. *We denote the set of all* (r,s)*-fuzzy open Q-neighborhood of* x_t *by* $\mathbf{Q}_{\tau,\tau^*}(x_t, r, s)$.

Definition 2.6. *Let* (X, τ, τ^*) *be an I-dfts,* $\lambda \in I^X$, $x_t \in \mathbf{P}(X)$, $r \in I_0$ *and* $s \in I_1$. x_t *is called* (r,s)*-fuzzy* θ*-cluster point of* λ *if for every* $\mu \in \mathbf{Q}_{\tau,\tau^*}(x_t, r, s)$, *we have* $C_{\tau,\tau^*}(\mu, r, s) q \lambda$. *We denote* $D_{\tau,\tau^*}(\lambda, r, s) = \bigvee\{x_t \in \boldsymbol{P}(X) \mid x_t$ is (r,s)*-fuzzy* θ*-cluster point of* $\lambda\}$. *Where* $D_{\tau,\tau^*}(\lambda, r, s)$ *is called* (r,s)*-fuzzy* θ*-closure of* λ.

Theorem 2.4. *Let* (X, τ, τ^*) *an I-dfts. For* λ, $\mu \in I^X$ *and* r, $s \in I_0$, *we have the following:*

(1) $D_{\tau,\tau^*}(\lambda, r, s) = \bigwedge\{\mu \in I^X \mid \lambda \leq I_{\tau,\tau^*}(\mu, r, s),\ \tau(\underline{1}-\mu) \geq r, \tau^*(\underline{1}-\mu) \leq s\}$,
(2) x_t *is* (r,s)*-fuzzy* θ*-cluster point of* λ *iff* $x_t \in D_{\tau,\tau^*}(\lambda, r, s)$.
(3) $C_{\tau,\tau^*}(\lambda, r, s) \leq D_{\tau,\tau^*}(\lambda, r, s)$,
(4) *If* $\tau(\lambda) \geq r$ and $\tau^*(\lambda) \leq s$, *then* $C_{\tau,\tau^*}(\lambda, r, s) = D_{\tau,\tau^*}(\lambda, r, s)$,
(5) *If* λ *is* (r,s)*-fpo, then* $C_{\tau,\tau^*}(\lambda, r, s) = D_{\tau,\tau^*}(\lambda, r, s)$,
(6) *If* λ *is* (r,s)*-fpo and* $\lambda = C_{\tau,\tau^*}(I_{\tau,\tau^*}(\lambda, r, s), r, s)$, *then* $D_{\tau,\tau^*}(\lambda, r, s) = \lambda$.

The complement of (r,s)-fuzzy θ-closed set is called (r,s)-fuzzy θ-open and the (r,s)-fuzzy θ-interior operator denoted by $T_{\tau,\tau^*}(\lambda, r, s)$ is defined by $T_{\tau,\tau^*}(\lambda, r, s) = \bigvee\{\nu \in I^X \mid C_{\tau,\tau^*}(\nu, r, s) \leq \lambda,\ \tau(\nu) \geq r, \tau^*(\nu) \leq s\}$.

Remark 2.1. From Theorem 2.4 It is easy to see that:

(1) $I_{\tau,\tau^*}(\lambda, r, s) \leq T_{\tau,\tau^*}(\lambda, r, s)$ for any $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$,
(2) $T_{\tau,\tau^*}(\lambda, r, s) = I_{\tau,\tau^*}(\lambda, r, s)$ for each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$; $\tau(\lambda) \geq r$ and $\tau^*(\lambda) \leq s$.

Double Fuzzy weakly preopen functions

Definition 3.7. *A function* $f : (X, \tau_1, \tau_1^*) \rightarrow (Y, \tau_2, \tau_2^*)$ *is said to be double fuzzy weakly preopen if*

$$f(\lambda) \leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\lambda, r, s)), r, s)$$

for each $\lambda \in I^X$, $r \in I_0$ *and* $s \in I_1$; $\tau_1(\lambda) \geq r$ *and* $\tau_1^*(\lambda) \leq s$.

Remark 3.2. Every double fuzzy weakly open function is double fuzzy preopen and every double fuzzy preopen function is double fuzzy weakly preopen, but the converse need not be true in general.

Example 3.1. *Let* $X = \{a, b, c\}$ *and* $Y = \{x, y, z\}$. *Fuzzy sets* λ_1, λ_2 *and* λ_3 *are defined as:*

$$\begin{array}{lll} \lambda_1(a) = 0.5, & \lambda_1(b) = 0.3, & \lambda_1(c) = 0.2, \\ \lambda_2(x) = 0.9, & \lambda_2(y) = 1, & \lambda_2(z) = 0.7, \\ \lambda_3(x) = 0.2, & \lambda_3(y) = 0.2, & \lambda_3(z) = 0.3. \end{array}$$

Define τ_1 *and* τ_2 *as follows:*

$$\tau_1(\lambda) = \begin{cases} 1 & \text{if} \quad \lambda = \underline{0}, \underline{1}; \\ \frac{1}{3} & \text{if} \quad \lambda = \lambda_1; \\ 0 & \text{otherwise.} \end{cases}, \quad \tau_1^*(\lambda) = \begin{cases} 0 & \text{if} \quad \lambda = \underline{0}, \underline{1}; \\ \frac{1}{4} & \text{if} \quad \lambda = \lambda_1; \\ 1 & \text{otherwise.} \end{cases}$$

$$\tau_2(\lambda) = \begin{cases} 1 & \text{if} \quad \lambda = \underline{0}, \underline{1}; \\ \frac{1}{3} & \text{if} \quad \lambda = \lambda_2; \\ \frac{2}{3} & \text{if} \quad \lambda = \lambda_3; \\ 0 & \text{otherwise.} \end{cases}, \quad \tau_2^*(\lambda) = \begin{cases} 0 & \text{if} \quad \lambda = \underline{0}, \underline{1}; \\ \frac{1}{4} & \text{if} \quad \lambda = \lambda_2; \\ \frac{1}{3} & \text{if} \quad \lambda = \lambda_3; \\ 1 & \text{otherwise.} \end{cases}$$

Then the mapping $f : (X, \tau_1, \tau_1^*) \rightarrow (Y, \tau_2, \tau_2^*)$ *defined by* $f(a) = z$, $f(b) = x$ *and* $f(c) = y$ *is double fuzzy weakly preopen but not double fuzzy preopen. Where* $\tau_1(\lambda) \geq \frac{1}{3}$, $\tau_1^*(\lambda) \leq \frac{1}{3}$ *and* $f(\lambda)$ *is not* $(\frac{1}{3}, \frac{1}{3})$*-fpo.*

Example 3.2. *Let* $X = \{a, b, c\}$ *and* $Y = \{x, y, z\}$. *Fuzzy sets* λ_1, λ_2 *and* λ_3 *are defined as:*

$$\begin{array}{lll} \lambda_1(a) = 0.5, & \lambda_1(b) = 0.3, & \lambda_1(c) = 0.2; \\ \lambda_2(x) = 0.9, & \lambda_2(y) = 1, & \lambda_2(z) = 0.7; \\ \lambda_3(x) = 0.2, & \lambda_3(y) = 0.9, & \lambda_3(z) = 0.3. \end{array}$$

Let (τ_1, τ_1^*) *and* (τ_2, τ_2^*) *defined as follows:*

$$\tau_1(\lambda) = \begin{cases} 1 & \text{if} \quad \lambda = \underline{0}, \underline{1} \\ \frac{1}{2} & \text{if} \quad \lambda = \lambda_1; \\ 0 & \text{otherwise.} \end{cases}, \quad \tau_1^*(\lambda) = \begin{cases} 0 & \text{if} \quad \lambda = \underline{0}, \underline{1} \\ \frac{1}{2} & \text{if} \quad \lambda = \lambda_1; \\ 1 & \text{otherwise.} \end{cases}$$

$$\tau_2(\lambda)=\begin{cases}1 & if \quad \lambda=\underline{0},\underline{1}\\ \frac{1}{2} & if \quad \lambda=\lambda_2;\\ \frac{1}{3} & if \quad \lambda=\lambda_3;\\ 0 & otherwise.\end{cases}, \quad \tau_2^*(\lambda)=\begin{cases}0 & if \quad \lambda=\underline{0},\underline{1}\\ \frac{1}{2} & if \quad \lambda=\lambda_2;\\ \frac{1}{3} & if \quad \lambda=\lambda_3;\\ 1 & otherwise.\end{cases}$$

Then the mapping $f:(X,\tau_1,\tau_1^*)\to(Y,\tau_2,\tau_2^*)$ defined by $f(a)=z$, $f(b)=x$ and $f(c)=y$ is double fuzzy weakly preopen but not double fuzzy weakly open. Since $f(\lambda_1)\not\leq I_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\lambda_1,r,s)),r,s)$.

Theorem 3.5. *For a function $f:(X,\tau_1,\tau_1^*)\to(Y,\tau_2,\tau_2^*)$. The following statements are equivalent:*

(1) f is double fuzzy weakly preopen,
(2) $f(T_{\tau_1,\tau_1^*}(\lambda,r,s))\leq PI_{\tau_2,\tau_2^*}(f(\lambda),r,s)$ for each $\lambda\in I^X$, $r\in I_0$ and $s\in I_1$,
(3) $T_{\tau_1,\tau_1^*}(f^{-1}(\nu),r,s)\leq f^{-1}(PI_{\tau_2,\tau_2^*}(\nu,r,s))$ for each $\nu\in I^Y$, $r\in I_0$ and $s\in I_1$,
(4) $f^{-1}(PC_{\tau_2,\tau_2^*}(\nu,r,s))\leq D_{\tau_1,\tau_1^*}(f^{-1}(\nu),r,s)$ for each $\nu\in I^Y$, $r\in I_0$ and $s\in I_1$.

Proof. (1) $\Rightarrow$ (2) Let $\lambda\in I^X$ and $x_p\in T_{\tau_1,\tau_1^*}(\lambda,r,s)$. Then there exists $\gamma\in\mathbf{Q}_{\tau_1,\tau_1^*}(x_p,r,s)$ such that $\gamma\leq C_{\tau_1,\tau_1^*}(\gamma,r,s)\leq\lambda$. Thus $f(\gamma)\leq f(C_{\tau_1,\tau_1^*}(\gamma,r,s))\leq f(\lambda)$ and hence

$$\begin{aligned}PI_{\tau_2,\tau_2^*}(f(\gamma),r,s)&\leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\gamma,r,s)),r,s)\\&\leq PI_{\tau_2,\tau_2^*}(f(\lambda),r,s).\end{aligned}$$

Since f is double fuzzy weakly preopen,

$$f(\gamma)\leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\gamma,r,s)),r,s)\leq PI_{\tau_2,\tau_2^*}(f(\lambda),r,s).$$

and hence $f(x_p)\in PI_{\tau_2,\tau_2^*}(f(\lambda),r,s)$. This shows that $x_p\in f^{-1}(PI_{\tau_2}(f(\lambda),r,s))$. Thus $T_{\tau_1}(\lambda,r,s)\leq f^{-1}(PI_{\tau_2,\tau_2^*}(f(\lambda),r,s))$ and so, $f(T_{\tau_1,\tau_1^*}(\lambda,r,s))\leq PI_{\tau_2,\tau_2^*}(f(\lambda),r,s)$.

(2) $\Rightarrow$ (1) Let $\mu\in I^X$; $\tau_1(\mu)\geq r$ and $\tau_1^*(\mu)\leq s$. Since $\mu\leq T_{\tau_1,\tau_1^*}(C_{\tau_1,\tau_1^*}(\mu,r,s),r,s)$, then

$$\begin{aligned}f(\mu)&\leq f(T_{\tau_1,\tau_1^*}(C_{\tau_1,\tau_1^*}(\mu,r,s),r,s))\\&\leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\mu,r,s)),r,s).\end{aligned}$$

Hence f is double fuzzy weakly preopen.

(2) $\Rightarrow$ (3) Let $\nu\in I^Y$. By using (2), $f(T_{\tau_1,\tau_1^*}(f^{-1}(\nu),r,s))\leq PI_{\tau_2,\tau_2^*}(\nu,r,s)$. Therefore, $T_{\tau_1,\tau_1^*}(f^{-1}(\nu),r,s)\leq f^{-1}(PI_{\tau_2,\tau_2^*}(\nu,r,s))$.

(3) $\Rightarrow$ (2) Trivial.

(3) $\Rightarrow$ (4) Let $\nu\in I^Y$. Using (3), we have

$$\begin{aligned}\underline{1}-D_{\tau_1,\tau_1^*}(f^{-1}(\nu),r,s)&=T_{\tau_1,\tau_1^*}(\underline{1}-f^{-1}(\nu),r,s)\\&=T_{\tau_1,\tau_1^*}(f^{-1}(\underline{1}-\nu),r,s)\\&\leq f^{-1}(PI_{\tau_2,\tau_2^*}(\underline{1}-\nu,r,s))\\&=f^{-1}(\underline{1}-PC_{\tau_2,\tau_2^*}(\nu,r,s))\\&=\underline{1}-(f^{-1}(PC_{\tau_2,\tau_2^*}(\nu,r,s))).\end{aligned}$$

Therefore, we obtain $f^{-1}(PC_{\tau_2,\tau_2^*}(\nu,r,s))\leq D_{\tau_1,\tau_1^*}(f^{-1}(\nu),r,s)$.

(4) $\Rightarrow$ (3) Similarly we obtain, $\underline{1}-f^{-1}(PI_{\tau_2,\tau_2^*}(\nu,r,s))\leq\underline{1}-T_{\tau_1,\tau_1^*}(f^{-1}(\nu),r,s)$, for every $\nu\in I^Y$, $r\in I_0$ and $s\in I_1$, i.e., $T_{\tau_1,\tau_1^*}(f^{-1}(\nu),r,s)\leq f^{-1}(PI_{\tau_2,\tau_2^*}(\nu,r,s))$. □

Theorem 3.6. *For the function $f:(X,\tau_1,\tau_1^*)\to(Y,\tau_2,\tau_2^*)$. The following statements are equivalent:*

(1) f is double fuzzy weakly preopen,
(2) For each $x_t\in\mathbf{P}(X)$ and each $\mu\in I^X$; $\tau_1(\mu)\geq r$ and $\tau_1^*(\mu)\leq s$ with $x_t\leq\mu$, there exists (r,s)-fpo set γ such that $f(x_t)\leq\gamma$ and $\gamma\leq f(C_{\tau_1,\tau_1^*}(\mu,r,s))$.

Proof. (1) $\Rightarrow$ (2) Let $x_t\in\mathbf{P}(X)$ and $\mu\in I^X$ such that $\tau_1(\mu)\geq r$, $\tau_1^*(\mu)\leq s$ and $x_t\leq\mu$. Since f is double fuzzy weakly preopen, then $f(\mu)\leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\mu,r,s)),r,s)$. Let $\gamma=PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\mu,r,s)),r,s)$. Hence $\gamma\leq f(C_{\tau_1,\tau_1^*}(\mu,r,s))$, with $f(x_t)\leq\gamma$.

(2) $\Rightarrow$ (1) Let $\mu\in I^X$; $\tau_1(\mu)\geq r$, $\tau_1^*(\mu)\leq s$ and $y_s\leq f(\mu)$. It follows from (2) that $\gamma\leq f(C_{\tau_1,\tau_1^*}(\mu,r,s))$ for some (r,s)-fpo $\gamma\in I^Y$ and $y_s\leq\gamma$. Hence we have, $y_s\leq\gamma\leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\mu,r,s)),r,s)$. This shows that $f(\mu)\leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\mu,r,s)),r,s)$, i.e. f is double fuzzy weakly preopen function. □

Theorem 3.7. *Let $f:(X,\tau_1,\tau_1^*)\to(Y,\tau_2,\tau_2^*)$ be a bijective function. Then the following statements are equivalent:*

(1) f is double fuzzy weakly preopen;
(2) $PC_{\tau_2,\tau_2^*}(f(\lambda),r,s)\leq f(C_{\tau_1,\tau_1^*}(\lambda,r,s))$ for each $\lambda\in I^X$, $r\in I_0$ and $s\in I_1$; $\tau_1(\lambda)\geq r$ and $\tau_1^*(\lambda)\leq s$;
(3) $PC_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(\nu,r,s)),r,s)\leq f(\nu)$ for each $\nu\in I^X$, $r\in I_0$ and $s\in I_1$; $\tau_1(\underline{1}-\nu)\geq r$ and $\tau_1^*(\underline{1}-\nu)\leq s$.

Proof. (1) $\Rightarrow$ (2) Let $\nu\in I^X$; $\tau_1(\nu)\geq r$ and $\tau_1^*(\nu)\leq s$. Then we have,

$$f(\underline{1}-\nu)=\underline{1}-f(\nu)\leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\underline{1}-\nu,r,s)),r,s),$$

and so $\underline{1}-f(\nu)\leq\underline{1}-PC_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(\nu,r,s)),r,s)$. Hence $PC_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(\nu,r,s)),r,s)\leq f(\nu)$.

(2) $\Rightarrow$ (3) Let $\lambda\in I^X$; $\tau_1(\lambda)\geq r$ and $\tau_1^*(\lambda)\leq s$. Since $C_{\tau_1,\tau_1^*}(\lambda,r,s)$ is (r,s)-fc set and $\lambda\leq I_{\tau_1,\tau_1^*}(C_{\tau_1,\tau_1^*}(\lambda,r,s),$

$r, s)$ by (3) we have $PC_{\tau_2,\tau_2^*}(f(\lambda), r, s) \leq PC_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(\lambda, r, s)), r, s) \leq f(C_{\tau_1,\tau_1^*}(\lambda, r, s))$.

(3) $\Rightarrow$ (2) Trivial.

(2) $\Rightarrow$ (1) Trivial. □

Theorem 3.8. *For a function $f : (X, \tau_1, \tau_1^*) \rightarrow (Y, \tau_2, \tau_2^*)$. The following statements are equivalent:*

(1) f is double fuzzy weakly preopen;
(2) $f(I_{\tau_1,\tau_1^*}(\nu, r, s)) \leq PI_{\tau_2,\tau_2^*}(f(\nu), r, s)$ for each $\nu \in I^X$, $r \in I_0$ and $s \in I_1$; $\tau_1(\nu) \geq r$ and $\tau_1^*(\nu) \leq s$;
(3) $f(I_{\tau_1,\tau_1^*}(C_{\tau_1,\tau_1^*}(\lambda, r, s), r, s)) \leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\lambda, r, s)), r, s)$ for each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$; $\tau_1(\lambda) \geq r$ and $\tau_1^*(\lambda) \leq s$;
(4) $f(\lambda) \leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\lambda, r, s)), r, s)$, for each (r, s)-fpo set $\lambda \in I^X$;
(5) $f(\lambda) \leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\lambda, r, s)), r, s)$, for each (r, s)-fαo set $\lambda \in I^X$.

Proof. (1) $\Rightarrow$ (2) Let $\nu \in I^X$, $r \in I_0$ and $s \in I_1$; $\tau_1(\underline{1} - \nu) \geq r$ and $\tau_1^*(\underline{1} - \nu) \leq s$. By (1),

$$\begin{aligned} f(I_{\tau_1,\tau_1^*}(\nu, r, s)) &\leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(I_{\tau_1,\tau_1^*}(\nu, r, s), r, s)), r, s) \\ &\leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\nu, r, s)), r, s) \\ &= PI_{\tau_2,\tau_2^*}(f(\nu), r, s) \end{aligned}$$

(2) $\Rightarrow$ (3) It is clear.

(3) $\Rightarrow$ (4) Let λ be (r, s)-fpo set. Hence by (3),

$$\begin{aligned} f(\lambda) &\leq f(I_{\tau_1,\tau_1^*}(C_{\tau_1,\tau_1^*}(\lambda, r, s), r, s)) \\ &\leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\lambda, r, s)), r, s). \end{aligned}$$

(4) $\Rightarrow$ (5) and (5) $\Rightarrow$ (1) are clear. □

Definition 3.8. *A function $f : (X, \tau_1, \tau_1^*) \rightarrow (Y, \tau_2, \tau_2^*)$ is said to be double fuzzy strongly continuous, if $f(C_{\tau_1,\tau_1^*}(\lambda, r, s)) \leq f(\lambda)$ for each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$.*

Theorem 3.9. *If $f : (X, \tau_1, \tau_1^*) \rightarrow (Y, \tau_2, \tau_2^*)$ is double fuzzy weakly preopen and double fuzzy strongly continuous function, then f is double fuzzy preopen.*

Proof. Let $\lambda \in I^X$ such that $\tau_1(\lambda) \geq r$ and $\tau_1^*(\lambda) \leq s$. Since f is double fuzzy weakly preopen

$$f(\lambda) \leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\lambda, r, s)), r, s).$$

However, since f is double fuzzy strongly continuous, then $f(\lambda) \leq PI_{\tau_2,\tau_2^*}(f(\lambda), r, s)$ and therefore $f(\lambda)$ is (r, s)-fpo. □

Definition 3.9. *A function $f : (X, \tau_1, \tau_1^*) \rightarrow (Y, \tau_2, \tau_2^*)$ is said to be double fuzzy contra-preclosed if $f(\lambda)$ is (r, s)-fpo for each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$; $\tau_1(\underline{1} - \lambda) \geq r$ and $\tau_1^*(\underline{1} - \lambda) \leq s$.*

Theorem 3.10. *If $f : (X, \tau_1, \tau_1^*) \rightarrow (Y, \tau_2, \tau_2^*)$ is double fuzzy contra-preclosed, then f is double fuzzy weakly preopen function.*

Proof. Let $\lambda \in I^X$; $\tau_1(\lambda) \geq r$ and $\tau_1^*(\lambda) \leq s$. Then, we have

$$f(\lambda) \leq f(C_{\tau_1,\tau_1^*}(\lambda, r, s)) = PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\lambda, r, s)), r, s).$$

□

The converse of the above theorem need not be true in general as in the following Example.

Example 3.3. *Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. Define fuzzy sets λ_1, λ_2 as follows:*

$$\lambda_1(a) = 0, \quad \lambda_1(b) = 0.2, \quad \lambda_1(c) = 0.7;$$
$$\lambda_2(x) = 0, \quad \lambda_2(y) = 0.2, \quad \lambda_2(x) = 0.2.$$

Let (τ_1, τ_1^*) and (τ_2, τ_2^*) defined as follows:

$$\tau_1(\lambda) = \begin{cases} 1 & \text{if} \quad \lambda = \underline{1}, \underline{0}; \\ \frac{1}{3} & \text{if} \quad \lambda = \lambda_1; \\ 0 & \text{otherwise.} \end{cases}, \quad \tau_1^*(\lambda) = \begin{cases} 0 & \text{if} \quad \lambda = \underline{1}, \underline{0}; \\ \frac{1}{3} & \text{if} \quad \lambda = \lambda_1; \\ 1 & \text{otherwise.} \end{cases}$$

$$\tau_2(\lambda) = \begin{cases} 1 & \text{if} \quad \lambda = \underline{1}, \underline{0}; \\ \frac{1}{3} & \text{if} \quad \lambda = \lambda_2; \\ 0 & \text{otherwise.} \end{cases}, \quad \tau_2^*(\lambda) = \begin{cases} 0 & \text{if} \quad \lambda = \underline{1}, \underline{0}; \\ \frac{1}{3} & \text{if} \quad \lambda = \lambda_2; \\ 1 & \text{otherwise.} \end{cases}$$

Then the function $f : (X, \tau_1, \tau_1^*) \rightarrow (Y, \tau_2, \tau_2^*)$ defined as $f(a) = x$, $f(b) = y$ and $f(c) = z$ is double fuzzy weakly preopen but it isn't double fuzzy contra-preclosed.

Definition 3.10. *An I-dfts (X, τ, τ^*) is said to be (r, s)-fuzzy regular space if for each $\lambda \in I^X$; $\tau(\lambda) \geq r$ and $\tau^*(\lambda) \leq s$ is a union of (r, s)-fo sets $\mu_i \in I^X$ such that $C_{\tau,\tau^*}(\mu_i, r, s) \leq \lambda$ for each $i \in J$.*

Theorem 3.11. *Let (X, τ, τ^*) be (r, s)-regular fuzzy topological space. Then, $f : (X, \tau_1, \tau_1^*) \rightarrow (Y, \tau_2, \tau_2^*)$ is double fuzzy weakly preopen if and only if f is double fuzzy preopen.*

Proof. The sufficiency is clear. For the necessity, let $\lambda \in I^X$, $r \in I_0$, $s \in I_1$; $\lambda \neq \underline{0}$, $\tau_1(\lambda) \geq r$ and $\tau_1^*(\lambda) \leq s$. For each $x_t \leq \lambda$, let $x_t \leq \mu_{x_t} \leq C_{\tau_1,\tau_1^*}(\mu_{x_t}, r, s) \leq \lambda$. Hence we obtain that $\lambda = \bigvee\{\mu_{x_t} \mid x_t \leq \lambda\} = \bigvee\{C_{\tau_1,\tau_1^*}(\mu_{x_t}, r, s) \mid x_t \leq \lambda\}$ and,

$$\begin{aligned} f(\lambda) &= \bigvee\{f(\mu_{x_t}) \mid x_t \leq \lambda\} \\ &\leq \bigvee\{PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\mu_{x_t}, r, s)), r, s) \mid x_t \leq \lambda\} \\ &\leq PI_{\tau_2,\tau_2^*}(f(\bigvee\{C_{\tau_1,\tau_1^*}(\mu_{x_t}, r, s) \mid x_t\}), r, s) \\ &= PI_{\tau_2,\tau_2^*}(f(\lambda), r, s). \end{aligned}$$

Thus f is double fuzzy preopen. □

Theorem 3.12. *If $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ is double fuzzy almost open function, then it is double fuzzy weakly preopen.*

Proof. Let $\lambda \in I^X$; $\tau_1(\lambda) \geq r$ and $\tau_1^*(\lambda) \leq s$. Since f is double fuzzy almost open and $I_{\tau_1,\tau_1^*}(C_{\tau_1,\tau_1^*}(\lambda, r, s), r, s)$ is (r, s)-fro, then

$$I_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(C_{\tau_1,\tau_1^*}(\lambda, r, s), r, s)), r, s) = f(I_{\tau_1,\tau_1^*}(C_{\tau_1,\tau_1^*} \times (\lambda, r, s), r, s))$$

and hence

$$\begin{aligned} f(\lambda) &\leq f(I_{\tau_1,\tau_1^*}(C_{\tau_1,\tau_1^*}(\lambda, r, s), r, s) \\ &\leq I_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\lambda, r, s)), r, s) \\ &\leq PI_{\tau_2,\tau_2^*}(f(C_{\tau_1,\tau_1^*}(\lambda, r, s)), r, s). \end{aligned}$$

This shows that f is double fuzzy weakly preopen. □

Definition 3.11. *Let (X, τ, τ^*) be an I-dfts, $r \in I_0$ and $s \in I_1$. The two fuzzy sets λ, $\mu \in I^X$ are said to be (r, s)-fuzzy separated iff $\lambda\bar{q}C_{\tau,\tau^*}(\mu, r, s)$ and $\mu\bar{q}C_{\tau,\tau^*}(\lambda, r, s)$. A fuzzy set which cannot be expressed as a union of two (r, s)-fuzzy separated sets is said to be (r, s)-fuzzy connected.*

Definition 3.12. *Let (X, τ, τ^*) an I-dfts. The fuzzy sets $\lambda, \mu \in I^X$ such that $\lambda \neq \underline{0}$, $\mu \neq \underline{0}$, are said to be fuzzy (r, s)-pre-separated if $\lambda\bar{q}PC_{\tau,\tau^*}(\mu, r, s)$ and $\mu\bar{q}PC_{\tau,\tau^*}(\lambda, r, s)$ or equivalently if there exist two (r, s)-fpo sets ν, γ such that $\lambda \leq \nu$, $\mu \leq \gamma$, $\lambda\bar{q}\gamma$ and $\mu\bar{q}\nu$. An I-dfts which can not be expressed as a union of two fuzzy (r, s)-pre-separated sets is said to be fuzzy (r, s)-pre-connected space.*

Theorem 3.13. *If $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ is an injective double fuzzy weakly preopen and strongly double fuzzy continuous function from the space (X, τ_1, τ_1^*) onto an (r, s)-fuzzy pre-connected space (Y, τ_2, τ_2^*), then (X, τ_1, τ_1^*) is (r, s)-fuzzy connected.*

Proof. Let (X, τ_1, τ_1^*) be not (r, s)-fuzzy connected. Then there exist (r, s)-fuzzy separated sets β, $\gamma \in I^X$ such that $\beta \vee \gamma = \underline{1}$. Since β and γ are (r, s)-fuzzy separated, there exists λ, $\mu \in I^X$; $\tau_1(\lambda) \geq r$, $\tau_1(\mu) \geq r$ and $\tau_1^*(\lambda) \leq s$, $\tau_1^*(\mu) \leq s$ such that $\beta \leq \lambda$, $\gamma \leq \mu$, $\beta\bar{q}\mu$ and $\gamma\bar{q}\lambda$. Hence we have $f(\beta) \leq f(\lambda)$, $f(\gamma) \leq f(\mu)$, $f(\beta)\bar{q}f(\mu)$ and $f(\gamma)\bar{q}f(\lambda)$. Since f is double fuzzy weakly preopen and double fuzzy strongly continuous function, from Theorem 3.10 we have $f(\lambda)$ and $f(\mu)$ are (r, s)-fpo sets. Therefore, $f(\beta)$ and $f(\gamma)$ are (r, s)-fuzzy pre-separated and

$$\underline{1} = f(\underline{1}) = f(\beta \vee \gamma) = f(\beta) \vee f(\gamma)$$

which is contradiction with (Y, τ_2, τ_2^*) is (r, s)-fuzzy pre-connected. Thus (X, τ_1, τ_1^*) is (r, s)-fuzzy connected. □

Double Fuzzy weakly preclosed functions

Definition 4.13. *A function $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ is said to be double fuzzy weakly preclosed function if*

$$PC_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(\lambda, r, s)), r, s) \leq f(\lambda)$$

for each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$; $\tau_1(\underline{1} - \lambda) \geq r$ and $\tau_1^*(\underline{1} - \lambda) \leq s$.

Remark 4.3. Clearly, every double fuzzy preclosed function is double fuzzy weakly preclosed, but the converse need not be true in general, as the next example shows.

Example 4.4. *Let $X = \{a, b\}$ and $Y = \{x, y\}$. Fuzzy sets λ_1 and λ_2 are defined as:*

$$\lambda_1(x) = 0.4, \qquad \lambda_1(y) = 0.3;$$
$$\lambda_2(a) = 0.5, \qquad \lambda_2(b) = 0.6.$$

Let

$$\tau_1(\lambda) = \begin{cases} 1 & \text{if} \quad \lambda = \underline{1}, \underline{0}; \\ \frac{1}{2} & \text{if} \quad \lambda = \lambda_2; \\ 0 & \text{otherwise.} \end{cases}, \qquad \tau_1^*(\lambda) = \begin{cases} 0 & \text{if} \quad \lambda = \underline{1}, \underline{0}; \\ \frac{1}{2} & \text{if} \quad \lambda = \lambda_2; \\ 1 & \text{otherwise.} \end{cases}$$

$$\tau_2(\lambda) = \begin{cases} 1 & \text{if} \quad \lambda = \underline{1}, \underline{0}; \\ \frac{1}{2} & \text{if} \quad \lambda = \lambda_1; \\ 0 & \text{otherwise.} \end{cases}, \qquad \tau_2^*(\lambda) = \begin{cases} 0 & \text{if} \quad \lambda = \underline{1}, \underline{0}; \\ \frac{1}{2} & \text{if} \quad \lambda = \lambda_1; \\ 1 & \text{otherwise.} \end{cases}$$

Then the function $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ defined by $f(a) = x$, $f(b) = y$ is double fuzzy weakly preclosed but is not double fuzzy preclosed.

Theorem 4.14. *For a function $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$. The following statements are equivalent.*

(1) f is double fuzzy weakly preclosed;
(2) $PC_{\tau_2,\tau_2^*}(f(\lambda), r, s) \leq f(C_{\tau_1,\tau_1^*}(\lambda, r, s))$ for each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$; $\tau_1(\lambda) \geq r$ and $\tau_1^*(\lambda) \leq s$;
(3) $PC_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(\lambda, r, s), r, s) \leq f(\lambda)$ for each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$; $\tau_1(\underline{1} - \lambda) \geq r$ and $\tau_1^*(\underline{1} - \lambda) \leq s$;

(4) *$PC_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(\lambda,r,s),r,s) \le f(\lambda)$ for each (r,s)-fpc set $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$;*
(5) *$PC_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(\lambda,r,s),r,s) \le f(\lambda)$ for each (r,s)-fαc $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$.*

Proof. Straightforward. □

Theorem 4.15. *For a function $f : (X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$. The following statements are equivalent.*

(1) *f is double fuzzy weakly preclosed;*
(2) *$PC_{\tau_2,\tau_2^*}(f(\lambda),r,s) \le f(C_{\tau_1,\tau_1^*}(\lambda,r,s))$ for each (r,s)-fro set $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$;*
(3) *For each $\nu \in I^Y$, $\mu \in I^X$, $r \in I_0$ and $s \in I_1$; $\tau_1(\mu) \ge r$ and $\tau_1^*(\mu) \le s$ with $f^{-1}(\nu) \le \mu$, there exists (r,s)-fpo set $\gamma \in I^Y$ with $\nu \le \gamma$ and $f^{-1}(\gamma) \le C_{\tau_1,\tau_1^*}(\mu,r,s)$;*
(4) *For each fuzzy point $y_s \in \mathbf{P}(Y)$ and each $\mu \in I^X$, $r \in I_0$ and $s \in I_1$ such that $\tau_1(\mu) \ge r$ and $\tau_1^*(\mu) \le s$ with $f^{-1}(y_s) \le \mu$, there exists (r,s)-fpo set $\gamma \in I^Y$; $y_s \le \gamma$ and $f^{-1}(\gamma) \le C_{\tau_1,\tau_1^*}(\mu,r,s)$;*
(5) *$PC_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(C_{\tau_1,\tau_1^*}(\lambda,r,s),r,s)),r,s) \le f(C_{\tau_1,\tau_1^*}(\lambda,r,s))$ for each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$;*
(6) *$PC_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(D_{\tau_1,\tau_1^*}(\lambda,r,s),r,s)),r,s) \le f(D_{\tau_1,\tau_1^*}(\lambda,r,s))$ for each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$;*
(7) *$PC_{\tau_2,\tau_2^*}(f(\lambda),r,s) \le f(C_{\tau_1,\tau_1^*}(\lambda,r,s))$ for each (r,s)-fpo set $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$.*

Proof. We will prove (2) ⇒ (3) and (1) ⇒ (6).

(2) ⇒ (3) : Let $\nu \in I^Y$, $r \in I_0$, $s \in I_1$ and let $\mu \in I^X$; $\tau_1(\mu) \ge r$ and $\tau_1^*(\mu) \le s$ with $f^{-1}(\nu) \le \mu$. Then $f^{-1}(\nu)\bar{q}C_{\tau_1,\tau_1^*}(\underline{1} - C_{\tau_1,\tau_1^*}(\mu,r,s),r,s)$ and consequently, $\nu\bar{q}f(C_{\tau_1,\tau_1^*}(\underline{1} - C_{\tau_1,\tau_1^*}(\mu,r,s),r,s)$. Since $\underline{1} - C_{\tau_1,\tau_1^*}(\mu,r,s)$ is (r,s)-fro, $\nu\bar{q}PC_{\tau_2,\tau_2^*}(f(\underline{1} - C_{\tau_1,\tau_1^*}(\mu,r,s)),r,s)$ by (2). Let $\gamma = \underline{1} - PC_{\tau_2,\tau_2^*}(f(\underline{1} - C_{\tau_1,\tau_1^*}(\mu,r,s)),r,s)$. Then γ is (r,s)-fpo with $\nu \le \gamma$ and

$$\begin{aligned} f^{-1}(\gamma) &\le \underline{1} - f^{-1}(PC_{\tau_2,\tau_2^*}(\underline{1} - C_{\tau_1,\tau_1^*}(\mu,r,s),r,s)) \\ &\le \underline{1} - f^{-1}f(\underline{1} - C_{\tau_1,\tau_1^*}(\mu,r,s)) \\ &\le C_{\tau_1,\tau_1^*}(\mu,r,s). \end{aligned}$$

(1) ⇒ (6) : Let $\nu \in I^Y$, $r \in I_0$ and $s \in I_1$; $\tau_2(\underline{1} - \nu) \ge r$, $\tau_2^*(\underline{1} - \nu) \le s$ and $y_s \le \underline{1} - f(\nu)$. Since $f^{-1}(y_s) \le \underline{1} - \nu$, there exists (r,s)-fpo $\gamma \in I^Y$ with $y_s \le \gamma$ and $f^{-1}(\gamma) \le C_{\tau_1,\tau_1^*}(\underline{1} - \nu,r,s) = \underline{1} - I_{\tau_1,\tau_1^*}(\nu,r,s)$ by (6). Therefore $\gamma\bar{q}f(I_{\tau_1,\tau_1^*}(\nu,r))$, so that $y_s \le \underline{1} - PC_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(\nu,r,s)),r,s)$. □

Theorem 4.16. *If $f : (X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ is double fuzzy weakly preclosed, then for each $y_s \in \mathbf{P}(Y)$ and each $\mu \in \mathbf{Q}_{\tau_1,\tau_1^*}(f^{-1}(y_s),r,s)$, there exists (r,s)-fpo set $\gamma \in I^Y$; $\gamma \in \mathbf{Q}_{\tau_2,\tau_2^*}(y_s,r,s)$, such that $f^{-1}(\gamma) \le C_{\tau_1,\tau_1^*}(\mu,r,s)$.*

Proof. Let $\mu \in \mathbf{Q}_{\tau_1,\tau_1^*}(f^{-1}(y_s,r,s)$. Then $\mu(x) + s > 1$ and hence there exists $t \in (0,1)$ such that $\mu(x) > t > 1 - s$. Then $\mu \in \mathbf{Q}_{\tau_1,\tau_1^*}(f^{-1}(y_t),r,s)$. By Theorem 3.7-6 there exists (r,s)-fpo set $\gamma \in I^Y$; $y_t \le \gamma$ such that $f^{-1}(\gamma) \le C_{\tau_1,\tau_1^*}(\mu,r,s)$. Now, $\gamma(y) > t$ and hence $\gamma(y) > 1 - s$. Thus γ is (r,s)-fpo neighborhood of y_s. □

Definition 4.14. *Let (X,τ,τ^*) be an I-dfts. A fuzzy set $\lambda \in I^X$ is called (r,s)-fuzzy pre-Q-neighborhood of x_t if there exists (r,s)-fpo set $\mu \in I^X$ such that $x_t q\mu \le \lambda$. We denote the set of all (r,s)-fuzzy pre-Q-neighborhood of x_t by $\boldsymbol{PQ}_{\tau,\tau^*}(x_t,r,s)$.*

Theorem 4.17. *In an I-dfts (X,τ,τ^*). A fuzzy point $x_t \in PC_{\tau,\tau^*}(\lambda,r)$ if and only if for every $\mu \in \boldsymbol{PQ}_{\tau,\tau^*}(x_t,r,s)$, $\mu q\lambda$ is hold.*

Proof. Straightforward. □

Theorem 4.18. *If $f : (X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ is double fuzzy weakly preclosed and if for each $\nu \in I^X$, $r \in I_0$ and $s \in I_1$; $\tau_1(\underline{1}-\nu) \ge r$, $\tau_1^*(\underline{1}-\nu) \le s$ and each $f^{-1}(y_s) \le \underline{1}-\nu$ there exists $\mu \in \mathbf{Q}_{\tau_1,\tau_1^*}(f^{-1}(y_s),r,s)$ such that $f^{-1}(y_s) \le \mu \le C_{\tau_1,\tau_1^*}(\mu,r,s) \le \underline{1}-\nu$. Then f is double fuzzy preclosed.*

Proof. Let $\nu \in I^X$, $r \in I_0$ and $s \in I_1$; $\tau_1(\underline{1} - \nu) \ge r$, $\tau_1^*(\underline{1} - \nu) \le s$ and let $y_s \le \underline{1} - f(\nu)$. Then $f^{-1}(y_s) \le \underline{1} - \nu$, and hence there exists $\mu \in \mathbf{Q}_{\tau_1,\tau_1^*}(f^{-1}(y_s),r,s)$ such that $f^{-1}(y_s) \le \mu \le C_{\tau_1,\tau_1^*}(\mu,r,s) \le \underline{1} - \nu$. Since f is double fuzzy weakly preclosed by using Theorem 3.12, there exists (r,s)-fuzzy pre-Q-neighborhood $\gamma \in I^Y$ with $y_s \le \gamma$ and $f^{-1}(\gamma) \le C_{\tau_1,\tau_1^*}(\mu,r,s)$. Therefore, we obtain $f^{-1}(\gamma)\bar{q}\nu$ and hence $\gamma\bar{q}f(\nu)$, this shows that $y_s \notin PC_{\tau_2,\tau_2^*}(f(\nu),r,s)$. Therefore, $f(\nu)$ is (r,s)-fpc and f is double fuzzy preclosed function. □

Definition 4.15. *A function $f : (X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ is said to be double fuzzy contra-open (resp. double fuzzy contra-closed) if $\tau_2(\underline{1}-f(\lambda)) \ge r$ and $\tau_2^*(\underline{1}-f(\lambda)) \le s$(resp. $\tau_2(f(\lambda)) \ge r$ and $\tau_2^*(f(\lambda)) \le s$) for each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$; $\tau_1(\lambda) \ge r$ and $\tau_1^*(\lambda) \le s$ (resp. $\tau_1(\underline{1} - \lambda) \ge r$ and $\tau_1^*(\underline{1} - \lambda) \le s$).*

Theorem 4.19. *If $f : (X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ is double fuzzy contra-open, then f is double fuzzy weakly preclosed.*

Proof. Let $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$ such that $\tau_1(\underline{1} - \lambda) \ge r$ and $\tau_1^*(\underline{1} - \lambda) \le s$. Then,

$$PC_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(\lambda,r,s)),r,s) \leq f(I_{\tau_1,\tau_1^*}(\lambda,r,s)) \leq f(\lambda).$$

□

Theorem 4.20. *If $f:(X,\tau_1,\tau_1^*)\to(Y,\tau_2,\tau_2^*)$ is double fuzzy weakly preclosed, then for every $\nu\in I^Y$ and every $\lambda\in I^X$, $r\in I_0$, $s\in I_1$ such that $\tau_1(\lambda)\geq r$ and $\tau_1^*(\lambda)\leq s$ with $f^{-1}(\nu)\leq\lambda$, there exists (r,s)-fpc set $\gamma\in I^Y$ such that $\nu\leq\gamma$ and $f^{-1}(\gamma)\leq C_{\tau_1,\tau_1^*}(\lambda,r,s)$.*

Proof. Let $\nu\in I^Y$ and let $\lambda\in I^X$, $r\in I_0$ and $s\in I_1$ such that $\tau_1(\lambda)\geq r$ and $\tau_1^*(\lambda)\leq s$ with $f^{-1}(\nu)\leq\lambda$. Put $\gamma = PC_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(C_{\tau_1,\tau_1^*}(\lambda,r,s),r,s)),r,s)$, then γ is (r,s)-fpc set in I^Y such that $\nu\leq\gamma$ since $\nu\leq f(\lambda)\leq f(I_{\tau_1,\tau_1^*}(C_{\tau_1,\tau_1^*}(\lambda,r,s),r,s))\leq PC_{\tau_2,\tau_2^*}(f(I_{\tau_1,\tau_1^*}(C_{\tau_1,\tau_1^*}(\lambda,r,s),r,s)),r,s)=\gamma$. And since f is double fuzzy weakly preclosed, $f^{-1}(\gamma)\leq C_{\tau_1,\tau_1^*}(\lambda,r,s)$. □

Corollary 4.21. *If $f:(X,\tau_1,\tau_1^*)\to(Y,\tau_2,\tau_2^*)$ is double fuzzy weakly preclosed, then for every $y_s\in\mathbf{P}(Y)$ and every $\lambda\in I^X$, $r\in I_0$ and $s\in I_1$ such that $\tau_1(\lambda)\geq r$ and $\tau_1^*(\lambda)\leq s$ with $f^{-1}(y_s)\leq\lambda$, there exists (r,s)-fpc set $\gamma\in I^Y$; $y_s\leq\gamma$ such that $f^{-1}(\gamma)\leq C_{\tau_1,\tau_1^*}(\lambda,r,s)$.*

Definition 4.16. *A fuzzy set $\lambda\in I^X$ is called (r,s)-fuzzy θ-compact if for each family $\{\mu_i\mid i\in J\}$ in $\{\mu\in I^X\mid\mu\in\mathbf{Q}_{\tau,\tau^*}(\lambda,r,s)\}$ satisfy $(\bigvee_{i\in J}\mu_i)(x)\geq\lambda(x)$ for each $x\in X$, there exist a finite subset J_0 of J such that $\lambda\leq I_{\tau,\tau^*}(\bigvee\{C_{\tau,\tau^*}(\mu_i,r,s)\mid i\in J_0\},r,s)$.*

Theorem 4.22. *If $f:(X,\tau_1,\tau_1^*)\to(Y,\tau_2,\tau_2^*)$ is double fuzzy weakly preclosed with all fibers (r,s)-fuzzy θ-closed, then $f(\lambda)$ is (r,s)-fpc for each (r,s)-fuzzy θ-compact $\lambda\in I^X$, $r\in I_0$ and $s\in I_1$.*

Proof. Let λ be (r,s)-fuzzy θ-compact and let $y_s\leq\underline{1}-f(\lambda)$. Then $f^{-1}(y_s)\bar{q}\lambda$ and for each $x_t\leq\lambda$ there is $\mu_{x_t}\in\mathbf{Q}_{\tau_1,\tau_1^*}(x_t,r,s)$ with $x_t\leq\mu_{x_t}$ and $C_{\tau_1,\tau_1^*}(\mu_{x_t},r,s)\bar{q}f^{-1}(y_s)$. Clearly $\{\mu_{x_t}\mid x_t\leq\lambda,\ \mu_{x_t}\in\mathbf{Q}_{\tau_1,\tau_1^*}(\lambda,r,s)\}$ satisfy $(\bigvee_{i\in J}\mu_i)(x)\geq\lambda(x)$ for each $x\in X$ and since λ is (r,s)-fuzzy θ-compact, there is $\{\mu_{x_1},\mu_{x_2},\mu_{x_3},..,\mu_{x_n}\}\subseteq\{\mu_{x_t}\mid x_t\leq\lambda,\ \mu_{x_t}\in\mathbf{Q}_{\tau_1,\tau_1^*}(\lambda,r,s)\}$ such that $\lambda\leq I_{\tau_1,\tau_1^*}(\xi,r,s)$, where $\xi=\bigvee\{C_{\tau_1,\tau_1^*}(\mu_{x_i},r,s)\mid i=1,2,\ldots,n\}$. Since f is double fuzzy weakly preclosed, by using Theorem 3.12 there exists $\gamma\in\mathbf{PQ}_{\tau_1,\tau_1^*}(y_s,r,s)$ with

$$f^{-1}(y_s)\leq f^{-1}(\gamma)\leq C_{\tau_1,\tau_1^*}(\underline{1}-\xi,r,s)=\underline{1}-I_{\tau_1,\tau_1^*}(\xi,r,s) \leq\underline{1}-\lambda.$$

Therefore $y_s\leq\gamma$ and $\gamma\bar{q}f(\lambda)$. Thus $y_s\leq\underline{1}-PC_{\tau_2,\tau_2^*}(f(\lambda),r,s)$. Thus $f(\lambda)$ is (r,s)-fpc set. □

Definition 4.17. *Let (X,τ,τ^*) be an I-dfts. The fuzzy sets λ, $\mu\in I^X$ are (r,s)-fuzzy strongly separated if there exist $\nu,\gamma\in I^X$ such that $\tau(\nu)\geq r$ and $\tau^*(\nu)\leq s$, $\tau(\gamma)\geq r$ with $\lambda\leq\nu$, $\mu\leq\gamma$ and $C_{\tau,\tau^*}(\nu,r,s)\bar{q}C_{\tau,\tau^*}(\gamma,r,s)$.*

Definition 4.18. *An I-dfts (X,τ,τ^*) is called (r,s)-fuzzy pre T_2 if for each x_{t_1}, x_{t_2} with different supports there exists (r,s)-fpo sets λ, $\mu\in I^X$ such that $x_{t_1}\leq\lambda\leq x_{1-t_2}$, $x_{t_2}\leq\mu\leq x_{1-t_1}$ and $\lambda\bar{q}\mu$.*

Theorem 4.23. *If $f:(X,\tau_1,\tau_1^*)\to(Y,\tau_2,\tau_2^*)$ is double fuzzy weakly preclosed surjection and all fibers are (r,s)-fuzzy strongly separated, then (Y,τ_1,τ_1^*) is (r,s)-fuzzy pre-T_2.*

Proof. Let y_{s_1}, $y_{s_2}\in\mathbf{P}(Y)$ and let $\gamma,\nu\in I^X$, $r\in I_0$ and $s\in I_1$; $\tau_1(\gamma)\geq r$, $\tau_1^*(\gamma)\leq s$, $\tau_1(\nu)\geq r$ and $\tau_1(\nu)\leq s$ such that $f^{-1}(y_{s_1})\leq\gamma$ and $f^{-1}(y_{s_2})\leq\nu$ respectively with $C_{\tau_1,\tau_1^*}(\gamma,r,s)\bar{q}C_{\tau_1,\tau_1^*}(\nu,r,s)$. By using Theorem 3.12-4 there are (r,s)-fpo sets λ, $\mu\in I^Y$ such that $y_{s_1}\leq\lambda$ and $y_{s_2}\leq\mu$, $f^{-1}(\lambda)\leq C_{\tau_1,\tau_1^*}(\gamma,r,s)$ and $f^{-1}(\mu)\leq C_{\tau_1,\tau_1^*}(\nu,r,s)$. Therefore $\lambda\bar{q}\mu$, because $C_{\tau_1,\tau_1^*}(\gamma,r,s)\bar{q}C_{\tau_2,\tau_2^*}(\nu,r,s)$ and f is surjective. Thus (Y,τ_2,τ_2^*) is (r,s)-fuzzy pre-T_2. □

Definition 4.19. *an I-dfts (X,τ,τ^*) is said to be (r,s)-extremally disconnected if $\tau(C_{\tau,\tau^*}(\lambda,r,s))\geq r$ and $\tau^*(C_{\tau,\tau^*}(\lambda,r,s))\leq s$ for each $\lambda\in I^X$; $\tau(\lambda)\geq r$ and $\tau^*(\lambda)\leq s$.*

Definition 4.20. *an I-dfts (X,τ,τ^*) is said to be (r,s)-fuzzy almost compact if for each (r,s)-fuzzy open cover $\{\lambda_i\mid i\in J\}$ of X, there is a finite subset J_0 of J such that $\bigvee\{C_{\tau,\tau^*}(\lambda_i,r,s)\mid i\in J_0\}=\underline{1}$.*

Definition 4.21. *A fuzzy set λ in an I-dfts (X,τ,τ^*) is said to be (r,s)-fuzzy p-compact iff for each family of (r,s)-fpo sets $\{\mu_i\mid i\in J\}$ satisfies $(\bigvee_{i\in J}\mu_i)(x)=\lambda(x)$ for each $x\in X$. There exists finite subfamily J_0 of J such that $(\bigvee_{i\in J_0}PC_{\tau,\tau^*}(\mu_i,r,s))(x)\geq\lambda(x)$ for each $x\in X$.*

Theorem 4.24. *Let (X,τ_1,τ_1^*) be (r,s)-extremally disconnected I-dfts. Let $f:(X,\tau_1,\tau_1^*)\to(Y,\tau_2,\tau_2^*)$ be double fuzzy open and double fuzzy preclosed injective function such that $f^{-1}(y_s)$ is (r,s)-fuzzy almost compact for each $y_s\in\mathbf{P}(Y)$. If $\lambda\in I^Y$ is (r,s)-fuzzy P-compact. Then $f^{-1}(\lambda)$ is (r,s)-fuzzy almost compact.*

Proof. Let $\{\nu_j\mid æ\in J\}$ be (r,s)-fuzzy open cover of $f^{-1}(\lambda)$. Then for each $y_s\leq\lambda\wedge f(X)$, $f^{-1}(y_s)\leq\bigvee\{C_{\tau_1,\tau_1^*}(\nu_j,r,s)\mid æ\in J(y_s)\}=\gamma_{y_s}$, for some finite subfamily $J(y_s)$ of J. Since (X,τ_1,τ_1^*) is (r,s)-extremally disconnected each $\tau_1(C_{\tau_1,\tau_1^*}(\nu_j,r,s))\geq r$ and

$\tau_1^*(C_{\tau_1,\tau_1^*}(\nu_j, r, s)) \leq s$, hence $\tau_1(\gamma_{y_s}) \geq r$ and $\tau_1^*(\gamma_{y_s}) \leq s$. So by Corollary 4.21 there exists (r, s)-fpc set μ_{y_s}; $y_s \leq \mu_{y_s}$ such that $f^{-1}(\mu_{y_s}) \leq C_{\tau_1,\tau_1^*}(\gamma_{y_s}, r, s)$. Then, $\{\mu_{y_s} \mid y_s \leq \lambda \wedge f(X)\} \vee \{\underline{1} - f(X)\}$ is (r, s)-fuzzy preclosed cover of λ, $\lambda \leq \bigvee\{C_{\tau_2,\tau_1}(\mu_{y_s}, r, s) \mid y_s \leq \lambda \wedge f(X)\} \vee \{C_{\tau_2,\tau_2^*}(\underline{1} - f(X), r, s)\}$ for some finite fuzzy subset K of $\lambda \wedge f(X)$. Hence,

$$\begin{aligned} f^{-1}(\lambda) &\leq \bigvee_{y_s \in K} f^{-1}(C_{\tau_2,\tau_2^*}(\mu_{y_s}, r, s)) \\ &\quad \vee \{f^{-1}(C_{\tau_2,\tau_2^*}(\underline{1} - f(X), r, s))\} \\ &\leq \bigvee_{y_s \in K} C_{\tau_1,\tau_1^*}(f^{-1}(\mu_{y_s}), r, s) \\ &\quad \vee \{C_{\tau_1,\tau_1^*}(f^{-1}(\underline{1} - f(X)), r, s)\} \\ &\leq \bigvee_{y_s \in K} C_{\tau_1,\tau_1^*}(f^{-1}(\mu_{y_s}), r, s) \end{aligned}$$

so $f^{-1}(\lambda) \leq \bigvee_{æ \in J(y_s), y_s \in K} C_{\tau_1,\tau_1^*}(\nu_æ, r, s)$. Therefore $f^{-1}(\lambda)$ is (r, s)-fuzzy almost compact. □

Competing interests
The authors declare that they have no competing interests.

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